USER MEETING 2011

ON THE MODELING OF TURBULENCE FOR FLOW OVER COMPLEX TERRAINS

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OUTLINE OF THE PRESENTATION

- **Introduction**
  - Models of Turbulence
  - Turbulence Models for ABL flows
- **Mathematical Formulation**
  - Equations for k and ε/ω
- **Case Study - Bolund Experiment**
  - Results & Discussion
- **Conclusions**

MODELS OF TURBULENCE

- **Isotropic Turbulence**
  - Mixing Length Models
  - One Equation Models
- **Two Equation Models**
  - The widely used two equation models are:
    - k- ε models
    - k- ω models
- **Anisotropic Effects**
  - Reynolds Stress Model (RSM)
  - Large Eddy Simulation (LES)
- **Direct Numerical Simulation (DNS)**

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Turbulence Models for ABL flows

Of the various turbulence models available for turbulence closure, the two equation models are quite lucrative from a computational point of view.

The models are reasonably accurate and computationally less expensive.

WindSim solves the equations for turbulent kinetic energy (k) and turbulent dissipation rate (ε) / turbulent frequency (ω) to represent flow dynamics over complex terrains

MATHEMATICAL FORMULATION

Two Equation Models

Define the kinetic energy of the fluctuating motion as
$k = \frac{u'^2 + v'^2 + w'^2}{2}$

If we take the velocity fluctuation as $v' = \sqrt{k}$ then

the turbulent kinematic viscosity/eddy viscosity $\nu_t = l_m \sqrt{k}$

K – EQUATION

For kinetic energy of turbulence, a transport equation is derived in the form

$$\underbrace{\frac{\partial k}{\partial t}}_{\text{storage}} + \underbrace{\left(u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} + w\frac{\partial k}{\partial z}\right)}_{\text{convective transport of k}}$$

$$= \underbrace{\frac{\partial}{\partial x}\left\{\nu_l + \frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\nu_l + \frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right\} + \frac{\partial}{\partial z}\left\{\nu_l + \frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial z}\right\}}_{\text{diffusive transport of k}}$$

$$+ \underbrace{P_k}_{\text{rate of production of k}} + \underbrace{D_k}_{\text{rate of dissipation of k}} \qquad (1)$$

K – EQUATION

The production due to viscous dissipation of the mean flow is given as (in Cartesian tensor notation):

$$\underbrace{P_k}_{\text{rate of production of k}} = \nu_t \left[\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right] \frac{\partial \bar{u}_i}{\partial x_j}$$

The rate of destruction of turbulent kinetic energy is given by

$$D_k = -\varepsilon$$

The k- ε model is an equilibrium isotropic model where, the equilibrium between convective or diffusive transports and rates of production or destruction is considered.

K – EQUATION

In addition to the k-equation, a transport equation is derived for the turbulent dissipation rate, ε. Dimensionally, $\varepsilon \sim k/t_0$ where t_0 = turbulent time scale, Let l_0 = turbulent length scale and v_0 = turbulent velocity scale $\nu_t \sim v_0 l_0$ or l_0^2/t_0

$$\varepsilon \rightarrow k/t_0$$

$$v_0 \rightarrow \sqrt{k}$$

$$\nu_t \sim v_0 l_0 \rightarrow \left[\frac{l_0}{t_0}\right] l_0 \rightarrow \left[\frac{l_0^2}{t_0}\right] \rightarrow \left[\frac{l_0^2}{t_0^2}\right] t_0 = v_0^2 . t_0$$

Since $v_0^2 = k$ and $t_0 = k/\varepsilon$

$\nu_t \sim k^2/\varepsilon \rightarrow \boxed{\nu_t = C_\mu . \frac{k^2}{\varepsilon}}$ where C_μ is a constant and is equal to 0.09 in the case of a standard k-ε model

ε- Equation

The equation for the turbulent dissipation rate is

$$\underbrace{\frac{\partial \varepsilon}{\partial t}}_{\text{storage}} + \underbrace{\left(u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} + w\frac{\partial \varepsilon}{\partial z} \right)}_{\text{convective transport of } \varepsilon}$$

$$= \underbrace{\frac{\partial}{\partial x}\left\{ \nu_l + \frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x} \right\} + \frac{\partial}{\partial y}\left\{ \nu_l + \frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y} \right\} + \frac{\partial}{\partial z}\left\{ \nu_l + \frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z} \right\}}_{\text{diffusive transport of } \varepsilon}$$

$$+ \underbrace{P_\varepsilon}_{\text{rate of production of } \varepsilon} + \underbrace{D_\varepsilon}_{\text{rate of dissipation of } \varepsilon} \qquad (2)$$

ε- Equation

For the ε equation, the production and destruction terms are expressed as

$$P_{\varepsilon} = C_{\varepsilon 1}.\frac{\epsilon}{k}.\nu_t\left[\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right]\frac{\partial \bar{u}_i}{\partial x_j}$$

$$D_{\varepsilon} = \frac{\varepsilon}{t_0} = C_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

The k-ε models have five constants $\sigma_k, \sigma_\varepsilon, C_{\epsilon 1}, C_{\epsilon 2}, C_\mu$. The model is not valid where non equilibrium and anisotropic effects may be important

ω- Equation

The equation for the turbulent specific dissipation rate or turbulent frequency is

$$\underbrace{\frac{\partial \omega}{\partial t}}_{\text{storage}} + \underbrace{\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + w\frac{\partial \omega}{\partial z} \right)}_{\text{convective transport of } \omega}$$

$$= \underbrace{\frac{\partial}{\partial x}\left\{ \nu_l + \frac{\nu_t}{\sigma_\omega}\frac{\partial \omega}{\partial x} \right\} + \frac{\partial}{\partial y}\left\{ \nu_l + \frac{\nu_t}{\sigma_\omega}\frac{\partial \omega}{\partial y} \right\} + \frac{\partial}{\partial z}\left\{ \nu_l + \frac{\nu_t}{\sigma_\omega}\frac{\partial \omega}{\partial z} \right\}}_{\text{diffusive transport of } \omega}$$

$$+ \underbrace{P_\omega}_{\text{rate of production of } \omega} + \underbrace{D_\omega}_{\text{rate of dissipation of } \omega} \qquad (3)$$

ω- Equation

The turbulent specific dissipation rate or turbulent frequency can be expressed as

$$\omega = \frac{\varepsilon}{C_\mu k}$$

$$\nu_t = \alpha^* \frac{k}{\omega}$$

where, $\alpha^* = 1.0$ for high Reynolds number flows. The production and destruction terms for ω are given by

$$P_\omega = \alpha . \frac{\omega}{k} . \nu_t \left[\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right] \frac{\partial \bar{u}_i}{\partial x_j}$$

The dissipation/destruction of turbulent kinetic energy and turbulent frequency or turbulent specific dissipation rate in the k-ω equations are quite different from the k-ϵ equations

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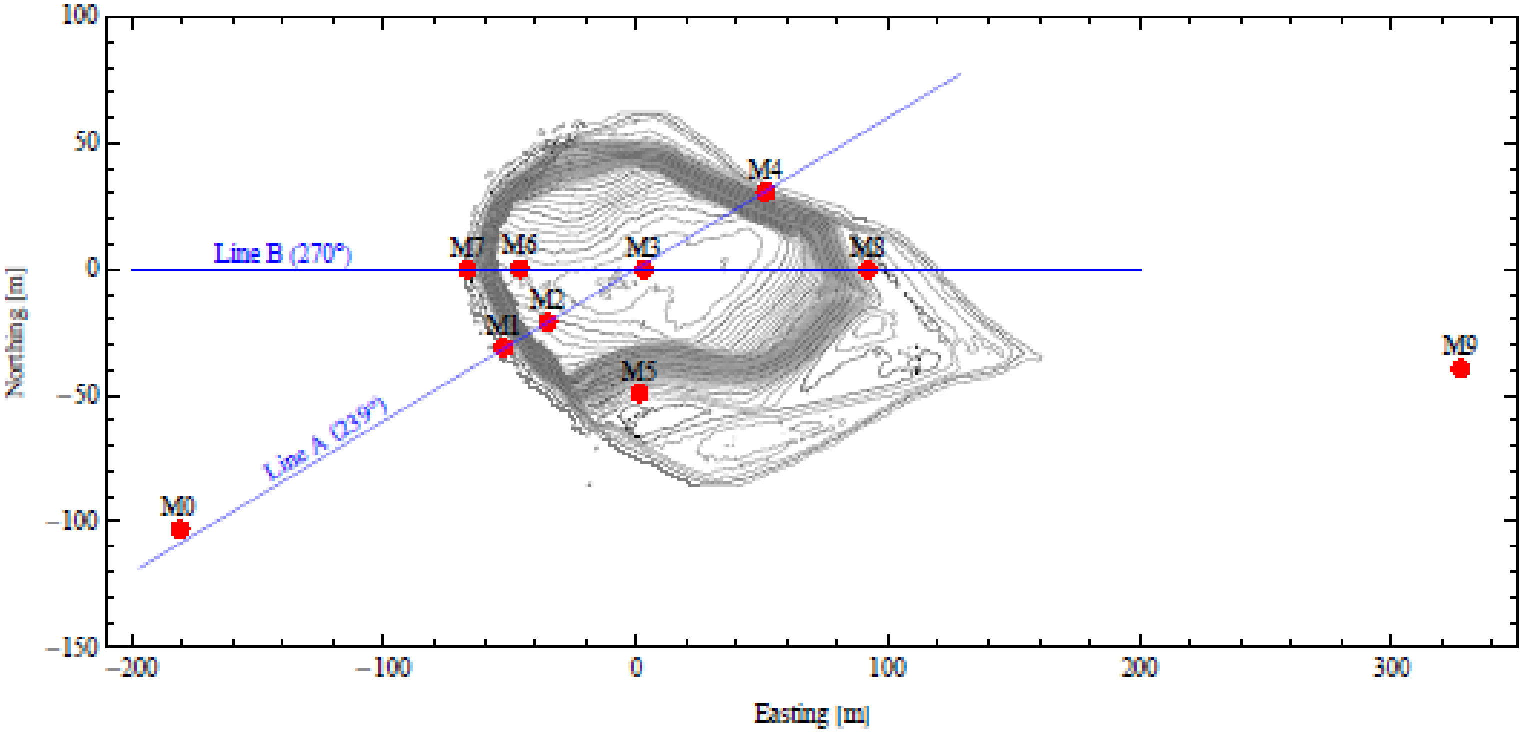
ω- Equation

The k-ω of Wilcox can handle recirculating flows quite efficiently as compared to the other two equation models. The model is well suited for flows with strong elliptic behavior where the flow region is dominated by far wakes and mixing layers.

CASE STUDY – BOLUND EXPERIMENT

Bolund orography and the position of the met masts



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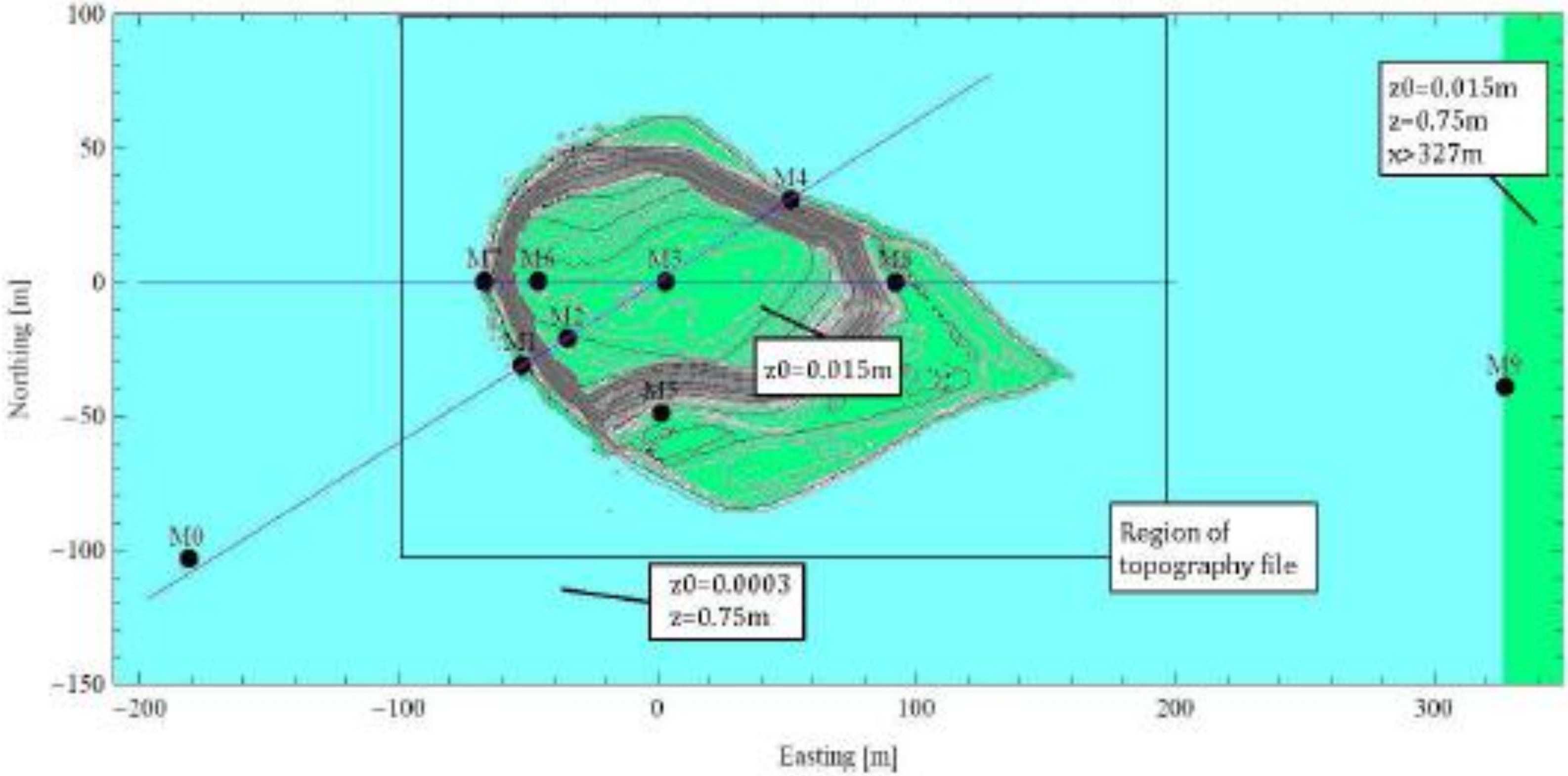
CASE STUDY – BOLUND EXPERIMENT



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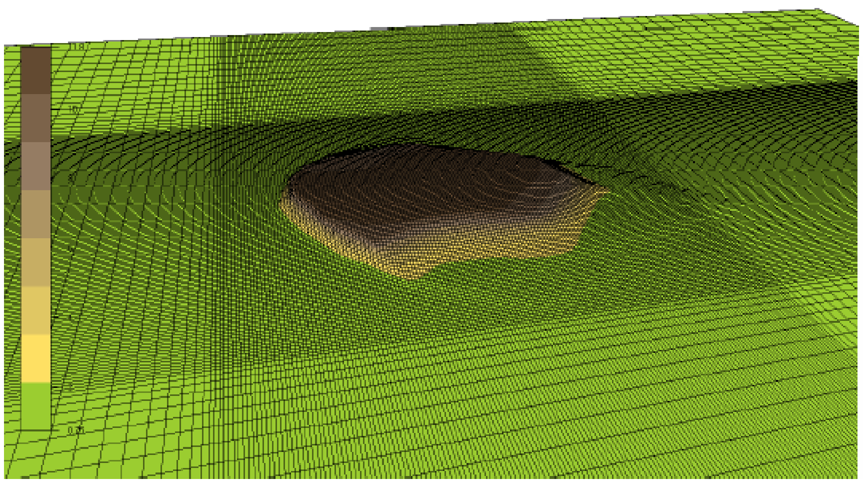
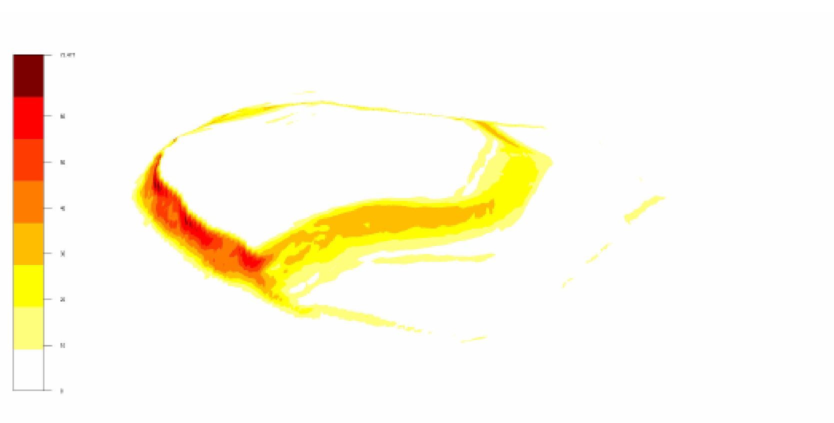
CASE STUDY – BOLUND EXPERIMENT

The Bolund hill considered for the present case study along with the body fitted coordinate (BFC) grid generated is shown in Fig. 1. 1.4 million cells are used for the present study.
Figure 2 shows the contours of inclination for the Bolund hill.





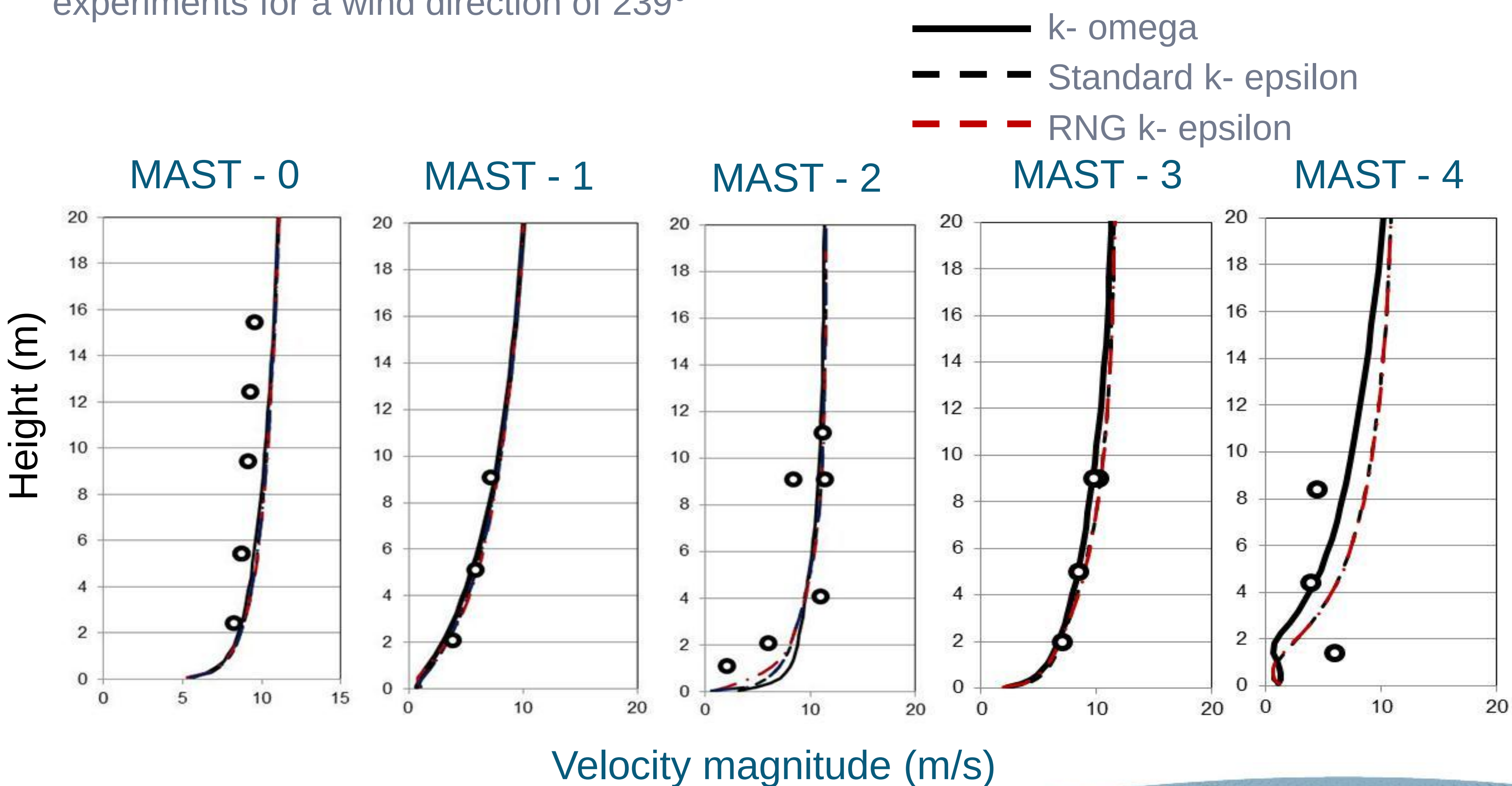
Simulation Parameters

- Two simulation cases are considered for the present investigation. The wind directions are 239° and 270 ° respectively.

- The well known logarithmic velocity profile is set for the whole computational domain and the turbulent kinetic energy cum dissipation rate / turbulent frequency are set as constants with height.

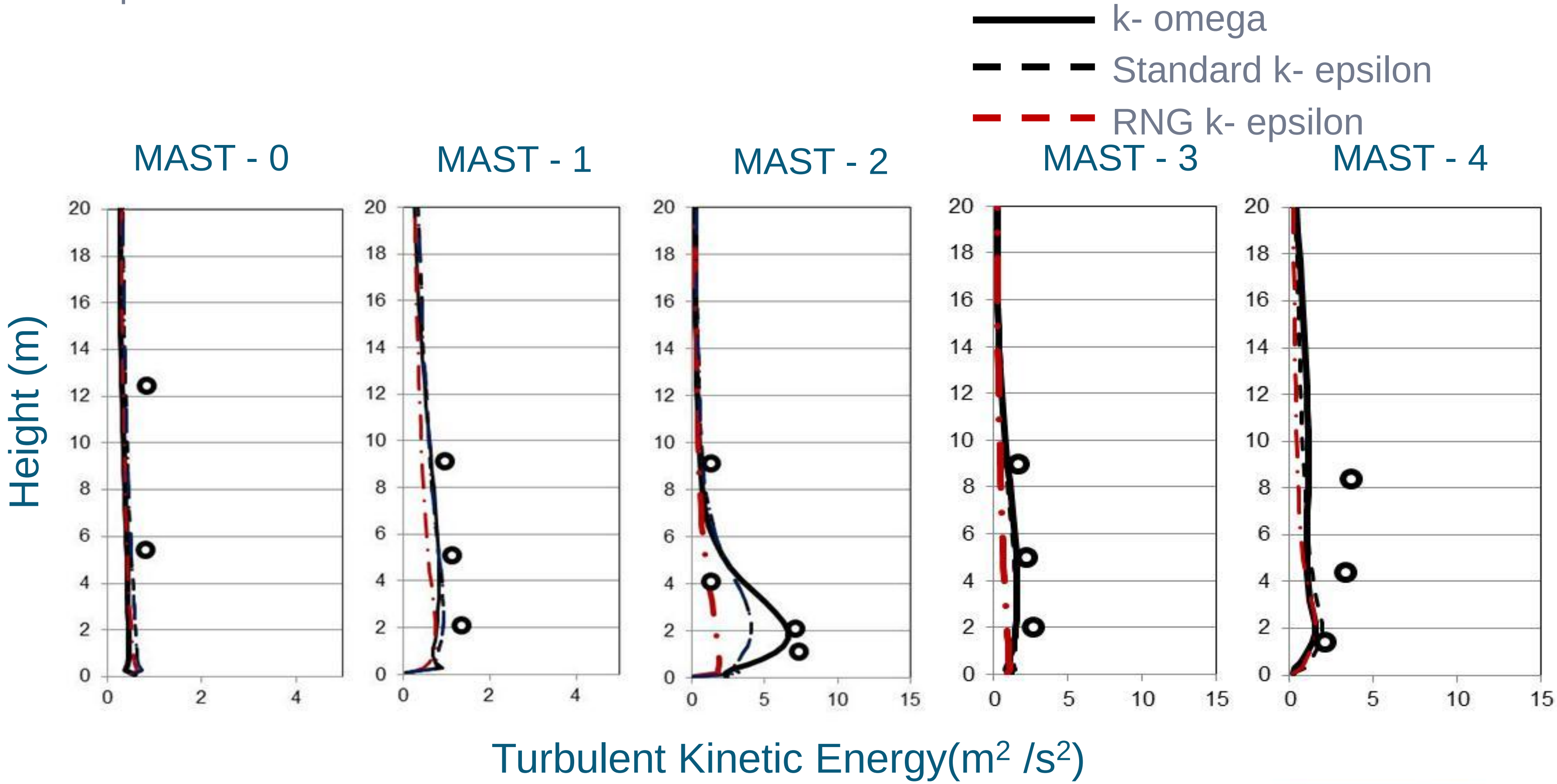
Results

Comparison of velocity magnitude obtained using the present simulations with experiments for a wind direction of 239°



Results

Comparison of turbulent kinetic energy obtained using the present simulations with experiments for a wind direction of 239°



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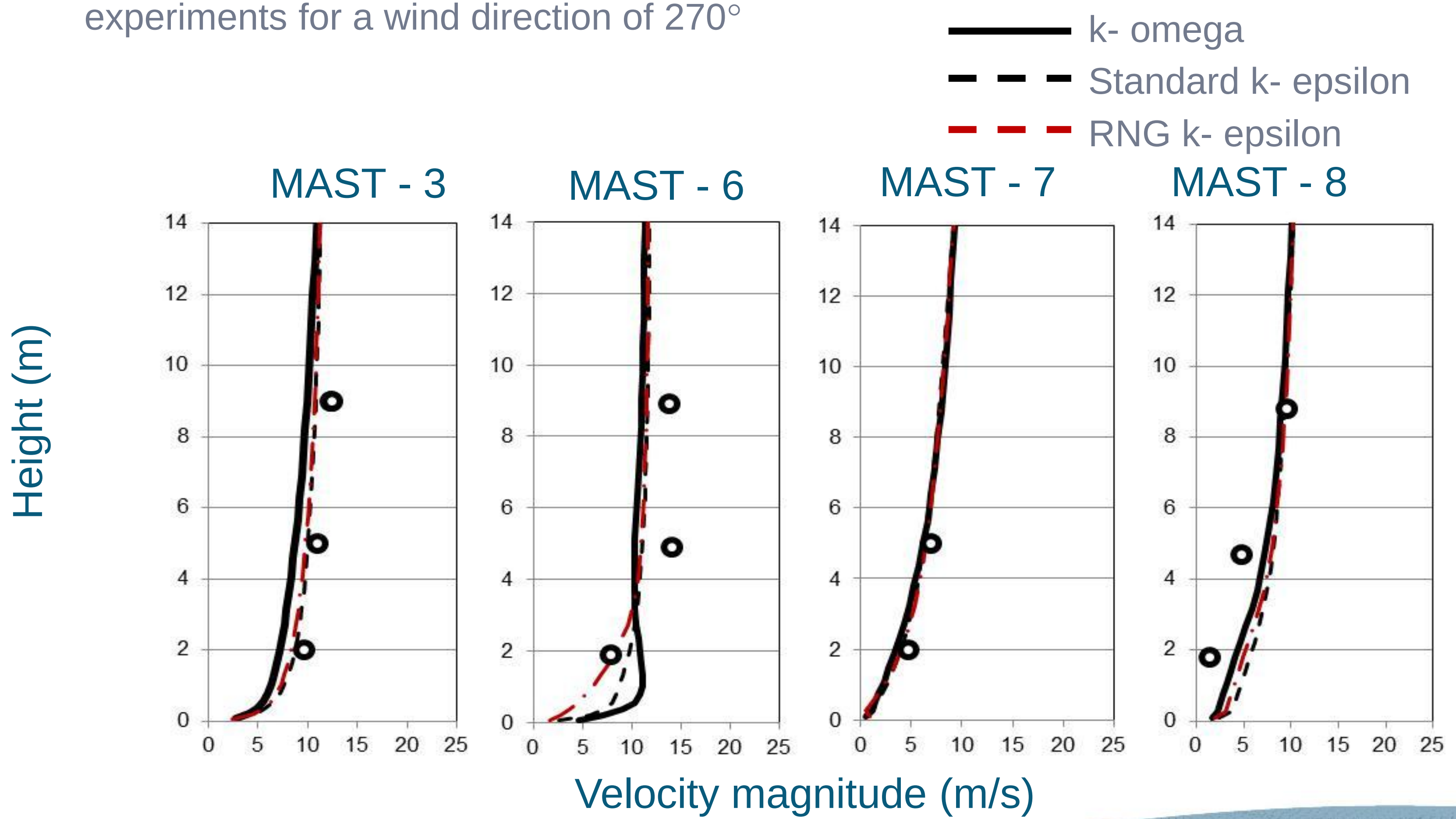
Results

Comparison of velocity magnitude obtained using the present simulations with experiments for a wind direction of 270°



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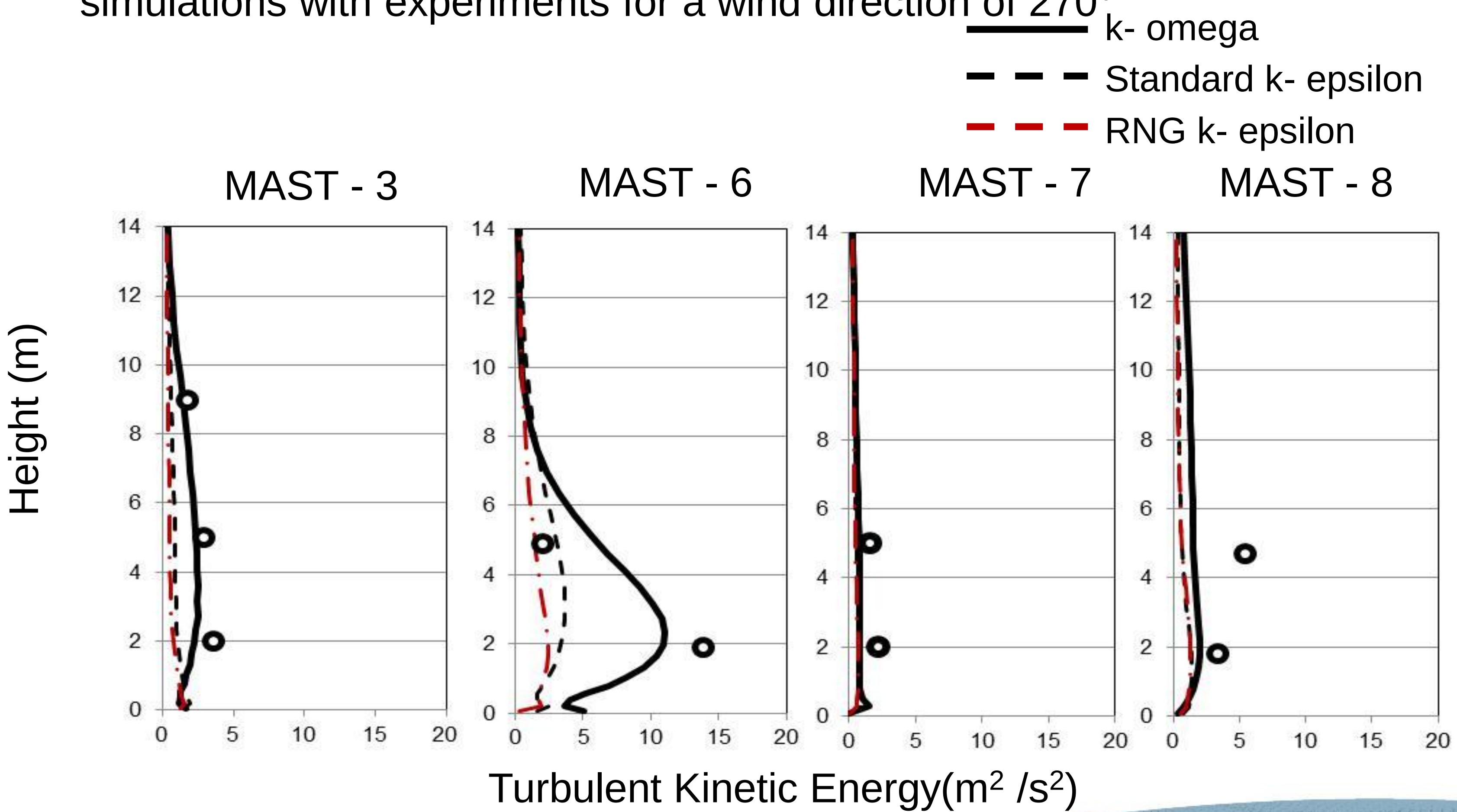
Comparison of turbulent kinetic energy obtained using the present simulations with experiments for a wind direction of 270°



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Conclusions

The following observations are made from the case study:

- K-ω model has been validated by comparing the numerical results from WindSim with the available experimental data.
- A comparison between the various two equation models shows that k-ω model is able to predict the mean velocity and the turbulent kinetic energy that are closer to the measurements.
- The Standard k-epsilon & RNG k-epsilon models predicts lower turbulence kinetic energy and thereby under predicts the turbulent intensity.
- The results obtained using the k-ω model of Wilcox are quite promising but more validation cases are required to confirm that the k-ω model is more suitable for ABL flows.